The Kelkar Education Trust's

Vinayak Ganesh Vaze College of Arts, Science & Commerce

(AUTONOMOUS)

College with Potential for Excellence

Mithaghar Road, Mulund East, Mumbai-400081, India

Phones :022-21631421, 221631423, 221631004 Fax : 022-221634262

email : vazecollege@gmail.com

Syllabus for S. Y. B. Sc. Programme: Mathematics

Syllabus as per **Choice Based Credit System**

(June 2020 Onwards)

Submitted by

Department of Mathematics

Vinayak Ganesh Vaze College of Arts, Science and Commerce

Mithagar Road, Mulund (East) Mumbai-400081. Maharashtra, India.

Tel: 022-21631004, Fax: 022-21634262

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(AUTONOMOUS)

❖ Syllabus as per Choice Based Credit System

<table>
<tr><td>1. Name of the Programme</td><td colspan="2">S. Y. B. Sc. Mathematics: CBCS</td></tr>
<tr><td colspan="3">The Mathematics course in S.Y.B.Sc is a one Year Full Time Course consisting of two semesters, to be known as Semester III and Semester IV. Each semester consists of THREE core courses and practicals.</td></tr>
<tr><td rowspan="5">2. Course Code</td><td>SEMESTER-III CODES</td><td>SEMESTER-IV CODES</td></tr>
<tr><td>SMAT301</td><td>SMAT401</td></tr>
<tr><td>SMAT302</td><td>SMAT402</td></tr>
<tr><td>SMAT303</td><td>SMAT403</td></tr>
<tr><td>SMATP301</td><td>SMATP401</td></tr>
<tr><td>3. Course Title</td><td colspan="2">MATHEMATICS</td></tr>
<tr><td>4. Semester wise Course Contents</td><td colspan="2">Copy of the detailed syllabus enclosed</td></tr>
<tr><td>5. References and additional references</td><td colspan="2">Enclosed in the Syllabus</td></tr>
<tr><td>6. No. of Credits per Semester</td><td colspan="2">09</td></tr>
<tr><td>7. No. of lectures per Unit</td><td colspan="2">15</td></tr>
<tr><td>8. No. of lectures per week</td><td colspan="2">09</td></tr>
<tr><td rowspan="2">9. No. of Practicals per week</td><td>For SMAT301 and SMAT302</td><td>01 (One Practical = 2 Lectures)</td></tr>
<tr><td>For SMAT303</td><td>01 (One Practical = 3 Lectures)</td></tr>
<tr><td rowspan="5">10. Scheme of Examination</td><td colspan="2">Semester End Exam:60 marks
(3 Questions of 20 marks each)</td></tr>
<tr><td colspan="2">Internal Assessment :40 marks</td></tr>
<tr><td colspan="2">Class Test :15 marks</td></tr>
<tr><td colspan="2">Project/ Assignment :15 marks</td></tr>
<tr><td colspan="2">Class Participation:10 marks</td></tr>
<tr><td>11. Special notes, if any</td><td colspan="2">No</td></tr>
<tr><td>12. Eligibility, if any</td><td colspan="2">As laid down in the College Admission brochure / website</td></tr>
<tr><td>13. Fee Structure</td><td colspan="2">As per College Fee Structure specifications</td></tr>
<tr><td>14. Special Ordinances / Resolutions, if any</td><td colspan="2">No</td></tr>
</table>

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Programme Structure and Course Credit Scheme:

Programme: S. Y. B. Sc.	Semester: III	Credits	Semester: IV	Credits
Course 1: Maths Paper-I	Course Code SMAT301	2	Course Code SMAT401	2
Course 2: Maths Paper-II	Course Code SMAT302	2	Course Code SMAT402	2
Course 3 : Maths Paper-III	Course Code SMAT303	2	Course Code SMAT403	2
Course 4: Practicals based on Maths paper I, II & III	Course Code SMATP301	3	Course Code SMATP401	3

Semester-wise Details of Mathematics Course

SEMESTER-III

Paper 1: CALCULUS III				
Course Code	**Unit**	**Topics**	**Credits**	**L/Week**
SMAT301	I	Riemann Integration	2	3
	II	Indefinite and Improper integrals		
	III	Beta and Gamma Functions and Applications		
Paper 2: ALGEBRA III				
SMAT302	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
Paper 3: DISCRETE MATHEMATICS				
SMAT303	I	Preliminary Counting	2	3
	II	Advanced Counting		
	III	Permutations and Recurrence Relations		
PRACTICALS				
SMATP301		Practicals based on SMAT301,SMAT302 and SMAT303	3	5

SEMESTER-IV

<table>
<tr><th colspan="5">Paper 1: CALCULUS IV</th></tr>
<tr><th>Course Code</th><th>Unit</th><th>Topics</th><th>Credits</th><th>L/week</th></tr>
<tr><td rowspan="3">SMAT401</td><td>I</td><td>Functions of several variables</td><td rowspan="3">2</td><td rowspan="3">3</td></tr>
<tr><td>II</td><td>Differentiation</td></tr>
<tr><td>III</td><td>Applications</td></tr>
<tr><th colspan="5">Paper 2: ALGEBRA IV</th></tr>
<tr><td rowspan="3">SMAT402</td><td>I</td><td>Quotient Spaces and Orthogonal Linear Transformations</td><td rowspan="3">2</td><td rowspan="3">3</td></tr>
<tr><td>II</td><td>Eigenvalues and Eigen Vectors</td></tr>
<tr><td>III</td><td>Diagonalization</td></tr>
<tr><th colspan="5">Paper 3: ORDINARY DIFFERENTIAL EQUATIONS</th></tr>
<tr><td rowspan="3">SMAT403</td><td>I</td><td>First Order First Degree Differential Equations</td><td rowspan="3">2</td><td rowspan="3">3</td></tr>
<tr><td>II</td><td>Second Order Differential Equations</td></tr>
<tr><td>III</td><td>Linear System of Ordinary Differential Equations</td></tr>
<tr><th colspan="5">PRACTICALS</th></tr>
<tr><td>SMATP401</td><td>--</td><td>Practicals based on SMAT401, SMAT402 and SMAT403</td><td>3</td><td>5</td></tr>
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<table>
<tr><th colspan="10">SEMESTER - III</th></tr>
<tr><th colspan="4">Teaching Scheme (Hrs/Week)</th><th colspan="3">Continuous Internal Assessment (CIA) 40 marks</th><th colspan="2">End Semester Examination Marks</th><th rowspan="2">Total</th></tr>
<tr><th>Course Code</th><th>L</th><th>P</th><th>C</th><th>CIA-1</th><th>CIA-2</th><th>CIA-3</th><th>Theory</th><th>Practical</th></tr>
<tr><td>SMAT301</td><td>03</td><td rowspan="2">01 (1P=2L)</td><td>2</td><td>15</td><td>15</td><td>10</td><td>60</td><td>--</td><td>100</td></tr>
<tr><td>SMAT302</td><td>03</td><td>2</td><td>15</td><td>15</td><td>10</td><td>60</td><td>--</td><td>100</td></tr>
<tr><td>SMAT303</td><td>03</td><td>01 (1P=3L)</td><td>2</td><td>15</td><td>15</td><td>10</td><td>60</td><td>--</td><td>100</td></tr>
<tr><td>SMATP301</td><td>--</td><td>--</td><td>3</td><td>--</td><td>--</td><td>--</td><td>--</td><td>150</td><td>150</td></tr>
<tr><td colspan="10">Total credits of the course = 02 + 02+ 02 + 03= 09</td></tr>
<tr><td colspan="10">Max. Time, End Semester Exam (Theory): 2 .00 Hrs.</td></tr>
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<table>
<tr><th colspan="11">SEMESTER - IV</th></tr>
<tr><th colspan="4">Teaching Scheme (Hrs/Week)</th><th colspan="3">Continuous Internal Assessment (CIA)
40 marks</th><th colspan="2">End Semester Examination Marks</th><th rowspan="2">Total</th></tr>
<tr><th>Course Code</th><th>L</th><th>P</th><th>C</th><th>CIA-1</th><th>CIA-2</th><th>CIA-3</th><th>Theory</th><th>Practical</th></tr>
<tr><td>SMAT401</td><td>03</td><td rowspan="2">01
(1P=2L)</td><td>2</td><td>15</td><td>15</td><td>10</td><td>60</td><td>--</td><td>100</td></tr>
<tr><td>SMAT402</td><td>03</td><td>2</td><td>15</td><td>15</td><td>10</td><td>60</td><td>--</td><td>100</td></tr>
<tr><td>SMAT403</td><td>03</td><td>01
(1P=3L)</td><td>2</td><td>15</td><td>15</td><td>10</td><td>60</td><td>--</td><td>100</td></tr>
<tr><td>SMATP401</td><td>--</td><td>--</td><td>3</td><td>--</td><td>--</td><td>--</td><td>--</td><td>150</td><td>150</td></tr>
<tr><td colspan="11">Total credits of the course = 02 + 02+ 02 + 03= 09</td></tr>
<tr><td colspan="11">Max. Time, End Semester Exam (Theory): 2 .00 Hrs.</td></tr>
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Course Content -Semester-III

<table>
<tr><th colspan="5">Paper 1: CALCULUS III</th></tr>
<tr><th>Course Code</th><th>Unit</th><th>Topics</th><th>Credits</th><th>L/Week</th></tr>
<tr><td rowspan="3">SMAT301</td><td>I</td><td>Riemann Integration</td><td rowspan="3">2</td><td rowspan="3">3</td></tr>
<tr><td>II</td><td>Indefinite and Improper integrals</td></tr>
<tr><td>III</td><td>Beta and Gamma Functions and Applications</td></tr>
<tr><th colspan="5">Paper 2: ALGEBRA III</th></tr>
<tr><td rowspan="3">SMAT302</td><td>I</td><td>Linear Transformations and Matrices</td><td rowspan="3">2</td><td rowspan="3">3</td></tr>
<tr><td>II</td><td>Determinants</td></tr>
<tr><td>III</td><td>Inner Product Spaces</td></tr>
<tr><th colspan="5">Paper 3: DISCRETE MATHEMATICS</th></tr>
<tr><td rowspan="3">SMAT303</td><td>I</td><td>Preliminary Counting</td><td rowspan="3">2</td><td rowspan="3">3</td></tr>
<tr><td>II</td><td>Advanced Counting</td></tr>
<tr><td>III</td><td>Permutations and Recurrence Relations</td></tr>
<tr><th colspan="5">PRACTICALS</th></tr>
<tr><td>SMATP301</td><td></td><td>Practicals based on SMAT301, SMAT302 and SMAT303</td><td>3</td><td>5</td></tr>
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<table>
<tr><td colspan="5">S. Y. B. Sc. MATHEMATICS: Choice Based Credit System</td></tr>
<tr><td colspan="5">Semester III</td></tr>
<tr><td colspan="5">PAPER: I - CALCULUS III</td></tr>
<tr><td colspan="3">Course Name: Calculus III (45 lectures)</td><td colspan="2">Course Code SMAT301</td></tr>
<tr><td colspan="3">Periods per week (1 period 48 minutes)</td><td colspan="2">03</td></tr>
<tr><td colspan="3">Credits</td><td colspan="2">02</td></tr>
<tr><td colspan="2" rowspan="3">Evaluation System</td><td></td><td>Hours</td><td>Marks</td></tr>
<tr><td>Theory Examination</td><td>2.0</td><td>60</td></tr>
<tr><td>Theory Internal</td><td></td><td>40</td></tr>
<tr><td>Unit No.</td><td colspan="3">Content</td><td>No. of lectures</td></tr>
<tr><td>Unit I</td><td colspan="3">Riemann Integration
Approximation of area, upper and lower Riemann sums and their properties, upper and lower integrals, definition of Riemann integral on a closed and bounded interval, Riemann criterion for integrability, if $a < c < b$ then
$f \in R[a,b]$, if and only if $f \in R[a,c]$ and $f \in R[c,b]$ and
$\int_a^b f = \int_a^c f + \int_c^b f$
Properties:
(i) If $f, g \in R[a,b] \Rightarrow f+g, \lambda f \in R[a,b]$
(ii) $\int_a^b (f+g) = \int_a^b f + \int_a^b g$
(iii) $\int_a^b \lambda f = \lambda \int_a^b f$
(iv) $f \in R[a,b] \Rightarrow |f| \in R[a,b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$
(v) $f \geq 0, f \in C[a,b] \Rightarrow f \in R[a,b]$
(vi) If f is bounded with finite number of discontinuities
then $f \in R[a,b]$, generalize this if f is monotone then $f \in R[a,b]$.</td><td>15</td></tr>
<tr><td>Unit II</td><td colspan="3">Indefinite and improper integrals
Continuity of $F(x) = \int_a^x f(t)dt$ where $f \in R[a,b]$
Fundamental theorem of calculus, Mean value theorem, Integration by parts, Leibnitz rule, Improper integrals-type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests.</td><td>15</td></tr>
<tr><td>Unit III</td><td colspan="3">Beta and Gamma Functions and Applications
β and Γ functions and their properties, relationship between β and Γ functions (without proof).
Applications of definite Integrals: Area between curves, finding volumes by slicing, volumes of solids of revolution-Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.</td><td>15</td></tr>
</table>

List of suggested practicals based on SMAT301:
1. Calculation of upper and lower sums, Problems based on definition of Riemann integral. 2. Properties of Riemann integral, Non-Riemann integrable functions. 3. Fundamental theorems of Calculus, Mean value theorems, integration by parts, Leibnitz rule. 4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, beta and gamma functions. 5. Finding area, volume, length. 6. Double integrals. 7. Miscellaneous theoretical questions based on three units.

Learning Outcomes:

On studying the syllabi the learner will be able to

- ✓ Know Riemann criterion for integrability
- ✓ Understand the properties of Riemann integrable functions.
- ✓ The applications of the fundamental theorems of integration.
- ✓ Understand Absolute & Conditional convergence of improper integrals
- ✓ Solve $\boldsymbol{\beta}$ and $\boldsymbol{\Gamma}$ functions and their properties
- ✓ Find Area and volume using integration.

Reference Books:

(1) Calculus Thomas Finney, ninth edition section 5.1, 5.2, 5.3, 5.4, 5.5, 5.6.

(2) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(3) Ajit Kumar, S.Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

(4) T. Apostol, Calculus Vol.2, John Wiley.

(5) K. Stewart, Calculus, Booke/Cole Publishing Co, 1994.

(6) J. E. Marsden, A.J. Tromba and A. Weinstein, Basic multivariable calculus.

(7) Bartle and Sherbet, Real analysis.

<table>
<tr><td colspan="5">S. Y. B. Sc. MATHEMATICS: Choice Based Credit System</td></tr>
<tr><td colspan="5">Semester III</td></tr>
<tr><td colspan="5">PAPER: II ALGEBRA III</td></tr>
<tr><td colspan="3">Course Name: Algebra III (45 lectures)</td><td colspan="2">Course Code SMAT302</td></tr>
<tr><td colspan="3">Lectures per week (1 period 48 minutes)</td><td colspan="2">03</td></tr>
<tr><td colspan="3">Credits</td><td colspan="2">02</td></tr>
<tr><td colspan="2" rowspan="3">Evaluation System</td><td></td><td>Hours</td><td>Marks</td></tr>
<tr><td>Theory Examination</td><td>2.0</td><td>60</td></tr>
<tr><td>Theory Internal</td><td></td><td>40</td></tr>
<tr><td>Unit No.</td><td colspan="3">Content</td><td>No. of lectures</td></tr>
<tr><td>Unit I</td><td colspan="3">Linear Transformations and Matrices
Linear transformations: Kernel, Image of a linear transformation, Rank T, Nullity T, and properties such as: for a linear transformation T, kernel (T) is a subspace of the domain space of T and the image (T) is a subspace of the co-domain space of T. If V, W are real vector spaces with $\{v_1, v_2, \dots \dots v_n\}$a basis of V and $\{w_1, w_2, \dots \dots w_n\}$any vectors in W then there exists a unique linear transformation $T: V \to W$ such that $T(v_j) = w_j \ \forall\ 1 \le j \le n$, Rank nullity theorem and examples. Linear isomorphisms, inverse of a linear isomorphism. Any n-dimensional real vector space is isomorphic to $\mathbb{R}^n$. Representation of linear maps by matrices and effect under a change of basis, examples.
Equivalence of rank of an mxn matrix A and rank of the linear transformation $L_A:\mathbb{R}^n \to \mathbb{R}^m (L_A(X) = AX)$. The dimension of solution space of the system of linear equations $AX = 0$ equals $n - rank(A)$. The solutions of non-homogeneous systems of linear equations represented by $AX = B$. Existence of a solution when rank(A)= rank(A: B)</td><td>15</td></tr>
<tr><td>Unit II</td><td colspan="3">Determinants
Definition of determinant as an n-linear skew-symmetric function from $\mathbb{R}^n$ X $\mathbb{R}^n$X$\mathbb{R}^n$× $\mathbb{R}^n \dots \times \mathbb{R}^n \to \mathbb{R}$ such that determinant of (E^1,E^2, ..., E^n) is 1, where E^j denotes the jth column of the n x n identity matrix I_n. Determinant of a matrix as determinant of its column vectors (or row vectors).
Existence and uniqueness of determinant function via permutations, Computation of determinant of 2x2, 3x3 matrices, diagonal matrices, Basic results on determinants such as $det(A^t) = det(A)$, $det(AB) = det(A)\, det(B)$, Laplace expansion of a determinant, determinant of upper triangular and lower triangular matrices, Vandermonde determinant.
Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants, the existence and uniqueness of the system $AX = B$, where A is an nxn matrix with $det(A) \neq 0$, $A^{-1} = \frac{1}{\det(A)} adj(A)$ for an invertible matrix A. Cramer's rule. Determinant as area and volume.</td><td>15</td></tr>
</table>

Unit III	**Inner Product Spaces** Dot product in $\mathbb{R}^n$. Definition of general inner product on a vector space over $\mathbb{R}$. Examples of inner product including the inner product $< f, g >= \int_{-\pi}^{\pi} f(t)g(t)dt$ on $C[-\pi, \pi]$, the space of continuous real valued functions on $[-\pi, \pi]$, Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in $\mathbb{R}^2$, Projections on a line, the projection being the closest approximation, orthogonal complements of a subspace, Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in $\mathbb{R}^3$ and $\mathbb{R}^4$.	**15**

List of suggested Practical for SMAT302:
1. Linear Transformation 2. System of linear equations 3. Determinants 4. Finding inverse of nxn matrices using adjoint ($n \leq 3$) 5. Inner product spaces, examples. Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$ 6. Gram-Schmidt method 7. Miscellaneous Theoretical Questions based on full paper

Learning Outcomes:

On studying the syllabi, the learner will be able to

- ♣ Understand the notion of Linear transformations & Rank nullity theorem
- ♣ Know the properties of linear transformation and isomorphism theorems
- ♣ Understand to solve determinant function by various methods
- ♣ Understand the Cramer's rule and able to find area and volume using determinants.
- ♣ Know the properties of inner product spaces
- ♣ Apply Cauchy-Schwarz inequality for obtaining orthonormal basis using Gram-Schmidt orthogonalization.

Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.

3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. BhimaSankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
6. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer Verlag New York.
7. Sheldon Axler, Linear Algebra done right, Springer Verlag.
8. Klaus Janich, Linear Algebra, Springer Verlag.
9. Otto Bretscher, Linear Algebra with Applications, Pearson Education.
10. Gareth Williams, Linear Algebra with Applications, Narosa Publication.

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<table>
<tr><th colspan="4">S. Y. B. Sc. MATHEMATICS : Choice Based Credit System</th></tr>
<tr><th colspan="4">Semester III</th></tr>
<tr><th colspan="4">PAPER : III DISCRETE MATHEMATICS</th></tr>
<tr><td colspan="2">Course Name: Discrete Mathematics (45 lectures)</td><td colspan="2">Course Code SMAT303</td></tr>
<tr><td colspan="2">Periods per week (1 period 48 minutes)</td><td colspan="2">03</td></tr>
<tr><td colspan="2">Credits</td><td colspan="2">02</td></tr>
<tr><td rowspan="3">Evaluation System</td><td></td><td>Hours</td><td>Marks</td></tr>
<tr><td>Theory Examination</td><td>2.0</td><td>60</td></tr>
<tr><td>Theory Internal</td><td></td><td>40</td></tr>
</table>

<table>
<tr><th>Unit No.</th><th>Content</th><th>No. of lectures</th></tr>
<tr><td>Unit I</td><td>Preliminary Counting
1. Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$, (0, 1), $\mathbb{R}$.
2. Addition and multiplication Principle, counting sets of pairs, two ways counting.
3. Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1,2,\ldots,n$.
4.Pigeonhole principle and its strong form, its applications to geometry, monotonic sequences etc.</td><td>15</td></tr>
<tr><td>Unit II</td><td>Advanced Counting
1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.
• $\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k}=\binom{m+n}{r}$
• $\sum_{k=0}^{n}\binom{i}{r}=\binom{n+1}{r+1}$
• $\sum_{i=0}^{k}\binom{k}{i}^2=\binom{2k}{k}$</td><td>15</td></tr>
</table>

	• $\sum_{i=0}^{n}\binom{n}{i}=2^n$ 2. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems. 3. Non-negative and positive solutions of equation $x_1+x_2+\cdots+x_k=n$ 4. Principle of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\varphi(n)$.	
Unit III	**Permutations and Recurrence Relation** 1. Permutation of objects, S_n, composition of permutations, definition of cycles, transposition, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, even and odd permutations, definition of A_n, signature of a permutation, cardinality of S_n and A_n.Order of elements of S_n 2. Recurrence Relations, definition of homogeneous, non-homogeneous, linear, nonlinear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous as well as non-homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method.	**15**

List of suggested practicals based on SMAT303
1.Derangement and rank signature of permutation. 2. Recurrence relation. 3. Problems based on counting principles, two way counting. 4. Stirling numbers of second kind, Pigeon hole principle. 5. Multinomial theorem, identities, permutation and combination of multi-set. 6. Inclusion-Exclusion principle. Euler phi function. 7. Miscellaneous theory questions from all units.

<u>Learning Outcomes:</u>

On studying the syllabi, the learner will be able to

- Know the basic facts about the cardinality of a set.
- Know about Stirling numbers & Pigeonhole principle
- Apply Binomial and Multinomial Theorem, Pascal identity while solving problems.
- Understand the Principle of inclusion and exclusion
- Understand elementary calculations in S_n
- Solve Recurrence Relations

Recommended Books:

1. Norman Biggs, Discrete Mathematics, Oxford University Press.
2. Richard Brualdi, Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy, Combinatorics-Theory and Applications, Affiliated East West Press.
4. S. S. Sane, Combinatorial Techniques, Hindustan Book Agency.
5. K. Rosen, Discrete Mathematics and its Applications, Tata McGraw Hills.
6. Schaum's outline series, Discrete mathematics.
7. Applied Combinatorics, Allen Tucker, John Wiley and Sons.
8. R. A. Beeler, How to count, Springer.

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Course Content -Semester-IV

Paper 1: CALCULUS IV				
Course Code	**Unit**	**Topics**	**Credits**	**L/week**
SMAT401	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
Paper 2: ALGEBRA IV				
SMAT402	I	Quotient Spaces and Orthogonal Linear Transformations	2	3
	II	Eigenvalues and Eigen Vectors		
	III	Diagonalization		
Paper 3: ORDINARY DIFFERENTIAL EQUATIONS				
SMAT403	I	First Order First Degree Differential Equations	2	3
	II	Second Order Differential Equations		
	III	Linear System of Ordinary Differential Equations		
PRACTICALS				
SMATP401	--	Practicals based on SMAT401, SMAT402 and SMAT403	3	5

<table>
<tr><th colspan="5">S. Y. B. Sc. MATHEMATICS :Choice Based Credit System</th></tr>
<tr><th colspan="5">Semester IV</th></tr>
<tr><th colspan="5">PAPER : I CALCULUS IV</th></tr>
<tr><td colspan="3">Course Name: Calculus IV (45 lectures)</td><td colspan="2">Course Code SMAT401</td></tr>
<tr><td colspan="3">Periods per week (1 period 48 minutes)</td><td colspan="2">03</td></tr>
<tr><td colspan="3">Credits</td><td colspan="2">02</td></tr>
<tr><td colspan="2" rowspan="3">Evaluation System</td><td></td><td>Hours</td><td>Marks</td></tr>
<tr><td>Theory Examination</td><td>2.0</td><td>60</td></tr>
<tr><td>Theory Internal</td><td></td><td>40</td></tr>
<tr><th>Unit No.</th><th colspan="3">Content</th><th>No. of lectures</th></tr>
<tr><td>Unit I</td><td colspan="3">Functions of Several Variables
Euclidean space $\mathbb{R}^n$, Euclidean norm function on $\mathbb{R}^n$, open ball and open sets in $\mathbb{R}^n$, sequences in $\mathbb{R}^n$, convergence of sequences and basic properties, subsequences (These concepts should be specifically discussed for $\mathbb{R}^2$ and $\mathbb{R}^3$). Functions from $\mathbb{R}^n$ to $\mathbb{R}$ (scalar fields) and from $\mathbb{R}^n$ to $\mathbb{R}^m$ (vector fields), limits and continuity of scalar fields and vector fields, basic results on algebra of limits and continuity, nonexistence of limits, relation between continuity of vector field and its component functions.
Directional Derivatives and Partial derivatives of scalar fields, higher order partial derivatives, gradient of a scalar field, mean value theorem for derivatives of scalar fields.</td><td>15</td></tr>
<tr><td>Unit II</td><td colspan="3">Differentiation
Differentiability of a scalar field at a point of $\mathbb{R}^n$ (in terms of linear transformation) and on open subsets of $\mathbb{R}^n$, the total derivative and its properties, uniqueness of total derivative of differentiable functions, differentiability of scalar field implies its continuity, necessary condition for differentiability, sufficient condition for differentiability, chain rule for derivatives of scalar fields, homogeneous functions and Euler's theorem, sufficient condition for equality of mixed partial derivatives (without proof).</td><td>15</td></tr>
<tr><td>Unit III</td><td colspan="3">Applications
Second order Taylor's formula for scalar fields. Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, differentiability of scalar field implies its continuity, chain rule for derivatives of vector fields (without proof).
Mean value inequality. Hessian matrix, Maxima, minima and saddle points, Second derivative test for extrema of functions of two variables. Method of Lagrange Multipliers.</td><td>15</td></tr>
</table>

List of Practicals based on SMAT401
1. Open sets in $\mathbb{R}^2$ and $\mathbb{R}^3$, sequences in $\mathbb{R}^2$ and $\mathbb{R}^3$, limits and continuity of scalar fields and vector fields, nonexistence of limits of scalar fields 2. Directional derivatives, partial derivatives and Mean value theorem 3. Total derivative of scalar fields, chain rules, Euler's theorem for homogeneous functions 4. Total derivative of vector fields, Jacobian matrix, chain rule for derivative of vector fields 5. Level sets, tangent planes, linear and quadratic approximations, Hessian matrix 6. Extreme values, saddle points and method of Lagrange's multipliers 7.Miscellaneous Theoretical Questions based on three units

Learning Outcomes:

On studying the syllabi the learner will be able to

- Learn conceptual variations while advancing from one variable to several variables in calculus
- Understand the notion of Limits, continuity in $\mathbb{R}^n$
- Find Differentiability of Scalar Field
- Apply Chain rule for derivatives, Euler's Theorem
- Find Differentiability of Vector fields
- Understand the Hessian matrix, Maxima, minima and saddle points

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Reference Books:

1. Tom M. Apostol, Calculus Vol. 2, second edition, John Wiley, India.
2. Jerrold E. Marsden, Anthony J. Tromba, Alan Weinstein, Basic Multivariable Calculus, Indian edition, Springer-Verlag.
3. Jerrold E. Marsden, Anthony J. Tromba, Vector Calculus, fifth edition, W.H. Freeman and Co, New York.
4. S.C. Malik, Savita Arora, Mathematical Analysis, third edition, New Age International Publishers, India.
5. D. Somasundaram, A Second Course in Mathematical Analysis, Narosa Publishing House, India.
6. Dennis G. Zill, Warren S. Wright, Calculus Early Transcendentals, fourth edition, Jones and Bartlett Publishers.
7. Sudhir R. Ghorpade, Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer.
8. Satish Shirali, Harkrishnan Lal Vasudeva, Multivariable Analysis, Springer.
9. William Trench, Introduction to Real Analysis, Free hyperlinked edition.

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S. Y. B. Sc. MATHEMATICS :Choice Based Credit System			
Semester IV			
PAPER : II ALGEBRA IV			
Course Name: Algebra IV (45 lectures)		**Course Code SMAT402**	
Periods per week (1 period 48 minutes)		**03**	
Credits		**02**	
Evaluation System		**Hours**	**Marks**
	Theory Examination	2.0	60
	Theory Internal		40

Unit No.	Content	No. of lectures
Unit I	**Quotient Spaces and Orthogonal Linear Transformations** Review of vector spaces over $\mathbb{R}$, subspaces and linear transformation. Quotient Spaces: For a real vector space V and a subspace W , the cosets $v + W$ and the quotient space V/W, First Isomorphism theorem of real vector spaces (fundamental theorem of homomorphism of vector spaces), Dimension and basis of the quotient space V/W , when V is finite dimensional. Orthogonal transformations: Isometries of a real finite dimensional inner product space, Translations and Reflections with respect to a hyperplane, Orthogonal matrices over $\mathbb{R}$, Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space, Orthogonal transformation of $\mathbb{R}$, Any orthogonal transformation in $\mathbb{R}$ is a reflection or a rotation, Characterization of isometries as composites of orthogonal transformations and translation. Characteristic polynomial of an nxn real matrix. Cayley Hamilton Theorem and its Applications (Proof assuming the result $A(adjA) = I_n$ for an $n \times n$ matrix over the polynomial ring $\mathbb{R}[t\ ]$.	**15**
Unit II	**Eigenvalues and eigen vectors** Eigen values and eigen vectors of a linear transformation $T : V \to V$, where V is a finite dimensional real vector space and examples, Eigen values and Eigen vectors of $n \times n$ real matrices, The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation. The characteristic polynomial of an nxn real matrix and a linear transformation of a finite dimensional real vector space to itself, characteristic roots, Similar matrices, Relation with change of basis, Invariance of the characteristic polynomial and (hence of the) eigenvalues of similar matrices, Every square matrix is similar to an upper triangular matrix. Minimal Polynomial of a matrix, Examples like minimal polynomial of scalar matrix, diagonal matrix, similar matrix, Invariant subspaces.	**15**

Unit III	**Diagonalization** Geometric multiplicity and Algebraic multiplicity of eigen values of an $n \times n$ real matrix, An $n \times n$ matrix A is diagonalizable if and only if it has a basis of eigenvectors of A if and only if the sum of dimension of eigen spaces of A is n if and only if the algebraic and geometric multiplicities of eigen values of A coincide, Examples of non diagonalizable matrices, Diagonalization of a linear transformation$T: V \rightarrow V$, where V is a finite dimensional real vector space and examples. Orthogonal diagonalisation and Quadratic Forms. Diagonalisation of real Symmetric matrices, Examples, Applications to real Quadratic forms, Rank and Signature of a Real Quadratic form, Classification of conics in $\mathbb{R}$ and quadric surfaces in $\mathbb{R}$. Positive definite and semi definite matrices, Characterization of positive definite matrices in terms of principal minors.	**15**

List of suggested practicals based on SMAT402:
1. Quotient Spaces, Orthogonal Transformations. 2. Cayley Hamilton Theorem and Applications 3. Eigen Values & Eigen Vectors of a linear Transformation/ Square Matrices 4. Similar Matrices, Minimal Polynomial, Invariant Subspaces 5. Diagonalisation of a matrix 6. Orthogonal Diagonalisation and Quadratic Forms. 7. Miscellaneous Theory Questions

Learning Outcomes:

On studying the syllabi the learner will be able to

- ✦ Grasp the concept of Quotient Spaces
- ✦ Understand the concept of Orthogonal transformations
- ✦ Relate matrices and linear transformations,
- ✦ Compute eigen values and eigen vectors of linear transformations.
- ✦ Understand Geometric multiplicity and Algebraic multiplicity
- ✦ Obtain various variants of diagonalization of linear transformations.

Recommended Books.

1. S. Kumaresan, Linear Algebra, A Geometric Approach.
2. Ramachandra Rao and P. Bhimasankaram, Tata McGraw Hill Publishing Company.

Additional Reference Books

1. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
2. L. Smith, Linear Algebra, Springer.
3. M. R. Adhikari and Avishek Adhikari, Introduction to linear Algebra, Asian Books Private Ltd.
4. K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
5. Inder K Rana, Introduction to Linear Algebra, Ane Books Pvt. Ltd.

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<table>
<tr><th colspan="5">S. Y. B. Sc. MATHEMATICS: Choice Based Credit System</th></tr>
<tr><th colspan="5">Semester IV</th></tr>
<tr><th colspan="5">PAPER: III ORDINARY DIFFERENTIAL EQUATIONS</th></tr>
<tr><td colspan="3">Course Name: Ordinary Differential Equations (45 lectures)</td><td colspan="2">Course Code SMAT403</td></tr>
<tr><td colspan="3">Periods per week (1 period 48 minutes)</td><td colspan="2">03</td></tr>
<tr><td colspan="3">Credits</td><td colspan="2">02</td></tr>
<tr><td rowspan="3" colspan="2">Evaluation System</td><td></td><td>Hours</td><td>Marks</td></tr>
<tr><td>Theory Examination</td><td>2.0</td><td>60</td></tr>
<tr><td>Theory Internal</td><td></td><td>40</td></tr>
<tr><th>Unit No.</th><th colspan="3">Content</th><th>No. of lectures</th></tr>
<tr><td>Unit I</td><td colspan="3">First Order First Degree Differential Equations
1. Definitions of: Differential Equation, Order and Degree of a differential Equation, Ordinary Differential Equation (ODE), Linear ODE, non-linear ODE.
2. Existence and uniqueness Theorem for the solution of a second order initial value problem (statement only). Definition of Lipschitz function. Examples based on verifying the conditions of existence and uniqueness theorem.
3. Review of solution of homogeneous and non-homogeneous linear differential equations of first order and first degree. Exact Equations: General Solution of Exact equations of first order and first degree, Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non-exact equations such as:
i) $\frac{1}{Mx+Ny}$ is an $I.F.$ if $Mx + Ny \neq 0$ and $Mx + Ny = 0$ is homogeneous.
ii) $\frac{1}{Mx-Ny}$ is an $I.F.$ if $Mx - Ny \neq 0$ and $Mx + Ny = 0$ is of the form $f_1(x,y)ydx + f_2(x,y)xdy = 0$.
iii) $e^{\int f(x)dx}$ (resp$e^{\int f(y)dy}$) is an $I.F.$ if $N \neq 0(resp\ M \neq 0$ and $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $\left(resp\ \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)\right)$ is a function of x $(resp\ y)$ alone, say $f(x)\big(resp\ g(y)\big)$.
Linear and reducible linear equations of first order. Finding solutions of first order differential equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.</td><td>15</td></tr>
</table>

Unit II	**Second Order Linear Differential Equations** 1. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals. 2. The homogeneous equation with constant coefficients, auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation. 3. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameter	**15**
Unit III	**Linear System of Ordinary Differential Equations** Existence and uniqueness theorems to be stated clearly when needed in the sequel. Study of homogeneous linear system of ODEs in two variables: Let $a_1(t), a_2(t), b_1(t), b_2(t)$ be continuous real valued functions defined on [a, b]. Fix $t_0 \in [a, b]$. Then there exists a unique solution x = x(t), y = y(t) valid throughout [a, b] of the following system: $\frac{dx}{dt} = a_1(t)x + b_1(t)y, \quad \frac{dy}{dt} = a_2(t)x + b_2(t)y,$ satisfying the initial conditions $x(t_0) = x_0$ & $y(t_0) = y_0$. The Wronskian W(t) of two solutions of a homogeneous linear system of ODEs in two variables, result: W (t) is identically zero or nowhere zero on [a, b]. Two linearly independent solutions and the general solution of a homogeneous linear system of ODEs in two variables. Explicit solutions of Homogeneous linear systems with constant coefficients in two variables, examples.	**15**

List of suggested practicals based on SMAT403:
1. Solving exact and non-exact ODEs 2. Linear and reducible to linear equations, applications to ODEs 3. Wronskian, Finding another linearly independent solution from known solution. 4. Finding general solution of homogeneous second order differential equations with constant coefficients 5. Method of undetermined coefficient 6. Method of Variation of parameters, qualitative properties of solutions 7. Miscellaneous theoretical questions from all units

Learning Outcomes:

On studying the syllabi the learner will be able to

- ❖ Understand the genesis of ordinary differential equations
- ❖ Learn various techniques of getting exact solutions of solvable first order differential equations and linear differential equations of higher order.
- ❖ Grasp the concept of a general solution of a linear differential equation of an arbitrary order and also learn a few methods to obtain the general solution of such equations.
- ❖ Understand Non-homogeneous equations
- ❖ Understand solving Homogeneous linear system of ODEs in two variables
- ❖ To find Wronskian W(t) of two solutions of a homogeneous linear system of ODEs in two variables

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Reference Books :

1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.
2. E. A. Codington, An introduction to ordinary differential equations, Dover Books.
3. S. L. Ross, Differential equations, 3rd edition, Wiley India Edition.
4. D. G. Zill, A first course in differential equations with modelling applications, 10th edition, Cengage Learning.

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THEORY EXAMINATION PATTERN

Que.1 A)	Attempt Any One:	(8 Marks)
	i) Theory Question based on Unit-I	
	ii) Theory Question based on Unit-I	
B)	Attempt Any Two:	(12 Marks)
	i) Problems based on Unit-I	
	ii) Problems based on Unit-I	
	iii) Problems based on Unit-I	
Que.2 A)	Attempt Any One:	(8 Marks)
	i) Theory Question based on Unit-II	
	ii) Theory Question based on Unit-II	
B)	Attempt Any Two:	(12 Marks)
	i) Problems based on Unit-II	
	ii) Problems based on Unit-II	
	iii) Problems based on Unit-II	
Que.3 A)	Attempt Any One:	(8 Marks)
	i) Theory Question based on Unit-III	
	ii) Theory Question based on Unit-III	
B)	Attempt Any Two:	(12 Marks)
	i) Problems based on Unit-III	
	ii) Problems based on Unit-III	
	iii) Problems based on Unit-III	

Semester End Examinations Practicals:

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses SMATP301, SMATP401.

Marks for Journals and Viva:

For each course SMAT301, SMAT302, SMAT303, SMAT401, SMAT402, SMAT403

1. Journals: 5 marks.

2. Viva: 5 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.

PRATICAL EXAMINATION PATTERN

Que.1	Attempt any 8 objectives out of 12 from the following:	(8 x 3=24 Marks)
Que.2	Attempt any two from the following:	(8 x 2 =16 Marks)
	a) Based on unit-I	
	b) Based on unit-II	
	c) Based on unit-III	

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